

Nonperturbative Phenomena and Phases of QCD

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1 Introduction

1.1 An outline

The lectures provide brief overview of what we have learned about QCD vacuum, hadrons and hadronic matter during the last 2 decades. Systematic description of the topics cover would need a large book, not lecture notes. Some material is available in reviews written in more technical way, for instantons and chiral symmetry breaking those are [1,2], for correlators, OPE etc [3]. A lot of other material is recent: one can only consult the original papers.

In these lectures there are not so many formulae. I tried to clarify the main physics point, the main questions debated today, and show few recent examples. Admittedly, the title of these lectures is very general: but they cover a lot of different phenomena. We will start with the QCD *vacuum structure*, move to *hadronic structure*, discuss *phases* of hot/dense QCD and eventually consider *high energy collisions* of hadrons and heavy ions.

The main line in all discussion would be a systematic use of semiclassical methods, specifically the instantons. The reasons for that are: (i) They are the only truly non-perturbative effects understood by now; (ii) They lead to large and probably even dominant effects in many cases; (iii) Due to progress during the last decade, we have near-quantitative theory of instanton effects, solved numerically to *all orders* in the so called 't Hooft interaction.

Although we still do not understand confinement, its companion problem - chiral symmetry breaking in the QCD vacuum - is now understood to a significant degree. Not only we have simple qualitative understanding of where those quasi-zero modes of the Dirac operator come from, but we can calculate their density, space-time shape and eventually **QCD correlation functions** with surprising accuracy. So, in a way, the problem of hadronic structure is nearly solved for light-quark hadrons¹.

As we will see below, the QCD phase diagram can also be well understood in the instanton framework. The boundaries of three basic phases of QCD: (i)

¹ Medium-heavy-quark ones, such as $\bar{c}c$, $\bar{b}b$ do care about confining potential, while very-heavy quarkonia need only the Coulomb forces.

hadronic phase, (ii) Quark-Gluon Plasma (QGP) and (iii) Color Superconductor (CS) phases appear as a balance between three basic pairing channels, being (i) attraction in scalar colorless $\bar{q}q$ channel; (ii) instanton-antiinstanton pairing induced by light quark exchanges; and (iii) attraction in scalar but colored qq channels.

The last part deals with heavy ion collisions: those are related to the rest of the lectures since this is how we try to access hot QCD experimentally. We will discuss first results coming from RHIC, show that matter produced seems to behave macroscopically (namely, hydrodynamically) with proper Equation of State. We will also try to connect rapid onset of QGP equilibration with existing perturbative and non-perturbative estimates.

1.2 Scales of QCD

Let me start with an introductory discussion of various “scales” of non-perturbative QCD. The major reason I do this is the following: some naive simplistic ideas we had in the early days of QCD, in the 70’s, are still alive today. I would strongly argue against the picture of non-perturbative objects as some structure-less fields with typical momenta of the order of $p \sim \Lambda_{QCD} \sim (1 \text{ fm})^{-1}$. In the mid-70’s people considered hadrons to be structure-less “bags” filled with near-massless perturbative quarks, with mild non-perturbative effects appearing at its boundaries and confining them at the scale of 1 fm.

One logical consequence of this picture would be applicability of the derivative expansion of the non-perturbative fields or Operator Product Expansion (OPE), the basis of the QCD sum rules. However, after the first successful applications of the method [4] rather serious problems[6] have surfaced. All spin-zero channels (as we will see, those are the ones directly coupled to instantons) related with quark or gluon-based operators alike, indicate unexpectedly large non-perturbative effects and deviate from the OPE predictions at very small distances.

It provided a very important lesson: *the non-perturbative fields form structures with sizes significantly smaller than 1 fm* and local field strength much larger than Λ^2 . Instantons are one of them: in order to describe many of these phenomena in a consistent way one needs instantons of small size [5] $\rho \sim 1/3 \text{ fm}$. We have direct confirmation of it from the lattice, but not real understanding of why there are no large-size instantons.

Furthermore, the instanton is not the only such small-scale gluonic object. We also learned from the lattice-based works that QCD flux tubes (or confining strings) also have small radius, only about $r_{string} \approx 1/5 \text{ fm}$. So, all hadrons (and clearly the QCD vacuum itself) have a *substructure*, with “constituent quarks” generated by instantons connected by such flux tubes.

Clearly this substructure should play an important role in hadronic physics. We would like to know why the usual quark model has been so successful in

spectroscopy, and why so little of exotic states have been seen. Also, high energy hadronic collisions must tell us a lot about substructure, since the famous Pomeron also belongs to a list of those surprisingly small non-perturbative objects.

At the opposite end of the spectrum, people have found that QCD seem to have also surprisingly *small energy/momentum scale*, several times lower than Λ . It was found that behavior of the so called “quenched” and true QCD is very different, but only if the quark mass is below some scale of the order of 20-50 MeV. As we will see below, this surprising low scale has been explained by properties of the instanton ensemble.

2 Chiral symmetry breaking and instantons

2.1 Brief history

Let me start around 1961, when the ideas about chiral symmetry and what it may take to break it spontaneously have appeared. The NJL model [12] was the first microscopic model which attempted to derive dynamically the properties of chiral symmetry breaking and pions, starting from some *hypothetical 4-fermion interaction*.

$$L_{NJL} = G(\pi^2 + \sigma^2) \quad (1)$$

where π, σ denote the corresponding scalar isovector and scalar isoscalar currents.

Let me make few comments about it.

- (i) It was the first bridge between the BCS theory of superconductivity and quantum field theory, leading the way to the Standard Model. It first showed that the vacuum can be truly nontrivial, a superconductor of a kind, with the mass gap $\Delta=330\text{--}400$ MeV, known as “constituent quark mass”.
- (ii) The NJL model has 2 parameters: the strength of its 4-fermion interaction G and the cutoff $\Lambda \sim .8 - 1\text{GeV}$. The latter regulates the loops (the model is non-renormalizable, which is OK for an effective theory) and is directly the “chiral scale” we are discussing. We will relate Λ to the typical instanton size ρ , and G to a combination $n\rho^2$ of the size and density of instantons.
- (iii) One non-trivial prediction of the NJL model was the mass of the scalar is $m_\sigma \approx 2m_{\text{const. quark}}$. Because this state is the P-wave in non-relativistic language, it means that there is strong attraction which is able to compensate exactly for rotational kinetic energy. For decades simpler hadronic models failed to get this effect, and even now spectroscopists still argue that this (40-year-old!) result is incorrect. However, lattice results in fact show that it is exactly right and theoretically understood by instantons. Moreover, the phenomenological sigma meson is being revived now, so possibly it will even get back to its proper place in Particle Data Table, after decades of absence.

Let me now jump to instantons. We will show below that they generate quite specific 4-fermion 't Hooft interaction [11] (for 2-flavor theory: for pedagogical reasons we ignore strange quarks altogether now). Furthermore, its Lagrangian includes the NJL one, but it also has 2 new terms:

$$L_{tHooft} = G(\pi^2 + \sigma^2 - \eta^2 - \delta^2) \quad (2)$$

with isoscalar pseudoscalar η and isovector scalar δ . T'Hooft's minus sign is crucial here: it shows that the axial U(1) symmetry (e.g. rotation of sigma into eta) is *not* a symmetry. That is why η (actually η' if strangeness is included) is *not* massless Goldstone particle like a pion.

The most important next development happened in 1980's: it has been shown in [5,13] that instanton-induced interaction does break *spontaneously* the $SU(N_f)$ chiral symmetry. Unlike the NJL model, the instanton-induced interaction has a natural cut-off parameter ρ , and the coupling constants are not free parameters, but determined by a physical quantity, the instanton density. That eventually allowed to solve in all orders in 't Hooft interaction, and get quantitative results, see [1].

2.2 General things about the instantons

I would omit from this paper general things about the instantons, well covered elsewhere. Let me just briefly mention that the topologically-nontrivial 4d solution was found by Polyakov and collaborators in [7], and soon it was interpreted as semi-classical tunneling between topologically non-equivalent vacua. The name itself was suggested by t Hooft, meaning "existing for an instant". Formally, instantons appear in the context of the semi-classical approximation to the (Euclidean) QCD partition function

$$Z = \int DA_\mu \exp(-S) \prod_f^{N_f} \det(\not{D} + m_f), \quad (3)$$

$$S = \frac{1}{4g^2} \int d^4x G_{\mu\nu}^a G_{\mu\nu}^a. \quad (4)$$

Here, S is the gauge field action and the determinant of the Dirac operator $\not{D} = \gamma_\mu(\partial_\mu - iA_\mu)$ accounts for the contribution of fermions. In the semi-classical approximation, we look for saddle points of the functional integral (3), i.e. configurations that minimize the classical action S . This means that saddle point configurations are solutions of the classical equations of motion.

These solutions can be found using the identity

$$S = \frac{1}{4g^2} \int d^4x \left[\pm G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \frac{1}{2} \left(G_{\mu\nu}^a \mp \tilde{G}_{\mu\nu}^a \right)^2 \right], \quad (5)$$

where $\tilde{G}_{\mu\nu} = 1/2\epsilon_{\mu\nu\rho\sigma}G_{\rho\sigma}$ is the dual field strength tensor (the field strength tensor in which the roles of electric and magnetic fields are reversed). Since

the first term is a topological invariant (see below) and the last term is always positive, it is clear that the action is minimal if the field is (anti) self-dual

$$G_{\mu\nu}^a = \pm \tilde{G}_{\mu\nu}^a. \quad (6)$$

The action of a self-dual field configuration is determined by its topological charge

$$Q = \frac{1}{32\pi^2} \int d^4x G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a. \quad (7)$$

From (5), we have $S = (8\pi^2|Q|)/g^2$. For finite action configurations, Q has to be an integer. The instanton is a solution with $Q = 1$ [7]

$$A_\mu^a(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2 + \rho^2}, \quad (8)$$

where the 't Hooft symbol $\eta_{a\mu\nu}$ is defined by

$$\eta_{a\mu\nu} = \begin{cases} \epsilon_{a\mu\nu} & \mu, \nu = 1, 2, 3, \\ & \delta_{a\mu}\nu = 4, \\ & -\delta_{a\nu}\mu = 4. \end{cases} \quad (9)$$

and ρ is an arbitrary parameter characterizing the size of the instanton. This original instanton has its non-trivial topology at large distances, but if we are to consider instanton ensemble, its another form, the so called *singular gauge* on is needed

$$A_\mu^a(x) = \frac{2\bar{\eta}_{a\mu\nu}x_\nu\rho^2}{(x^2 + \rho^2)x^2}, \quad (10)$$

because in this case the non-trivial topology is at the point singularity.

The classical instanton solution has a number of degrees of freedom, known as collective coordinates. In addition to the size, the solution is characterized by the instanton position z_μ and the color orientation matrix R^{ab} (corresponding to color rotations $A_\mu^a \rightarrow R^{ab}A_\mu^b$). A solution with topological charge $Q = -1$ can be constructed by replacing $\eta_{a\mu\nu} \rightarrow \bar{\eta}_{a\mu\nu}$, where $\bar{\eta}_{a\mu\nu}$ is defined by changing the sign of the last two equations in (9).

The physical meaning of the instanton solution becomes clear if we consider the classical Yang-Mills Hamiltonian (in the temporal gauge, $A_0 = 0$)

$$H = \frac{1}{2g^2} \int d^3x (E_i^2 + B_i^2), \quad (11)$$

where E_i^2 is the kinetic and B_i^2 the potential energy term. The classical vacua corresponds to configurations with zero field strength. For non-abelian gauge fields this limits the gauge fields to be “pure gauge” $A_i = iU(\mathbf{x})\partial_i U(\mathbf{x})^\dagger$. Such configurations are characterized by a topological winding number n_W which distinguishes between gauge transformations U that are not continuously connected.

This means that there is an infinite set of classical vacua enumerated by an integer n . Instantons are tunneling solutions that connect the different vacua. They have potential energy $B^2 > 0$ and kinetic energy $E^2 < 0$, their sum being zero at any moment in time. Since the instanton action is finite, the barrier between the topological vacua can be penetrated, and the true vacuum is a linear combination $|\theta\rangle = \sum_n e^{in\theta}|n\rangle$ called the theta vacuum. In QCD, the value of θ is an external parameter. If $\theta \neq 0$ the QCD vacuum breaks CP invariance. Experimental limits on CP violation require² $\theta < 10^{-9}$.

The rate of tunneling between different topological vacua is determined by the semi-classical (WKB) method. From the single instanton action one expects

$$P_{tunneling} \sim \exp(-8\pi^2/g^2). \quad (12)$$

The factor in front of the exponent can be determined by taking into account fluctuations $A_\mu = A_\mu^{cl} + \delta A_\mu$ around the classical instanton solution. This calculation was performed in a classic paper by 't Hooft [11]. The result is

$$dn_I = \frac{0.47 \exp(-1.68N_c)}{(N_c - 1)!(N_c - 2)!} \left(\frac{8\pi^2}{g^2} \right)^{2N_c} \exp\left(-\frac{8\pi^2}{g^2(\rho)}\right) \frac{d^4z d\rho}{\rho^5}, \quad (13)$$

where $g^2(\rho)$ is the running coupling constant at the scale of the instanton size. Taking into account quantum fluctuations, the effective action depends on the instanton size. This is a sign of the conformal (scale) anomaly in QCD. Using the one-loop beta function the result can be written as $dn_I/(d^4z) \sim d\rho \rho^{-5}(\rho \Lambda)^b$ where $b = (11N_c/3) = 11$ is the first coefficient of the beta function. Since b is a large number, small size instantons are strongly suppressed. On the other hand, there appears to be a divergence at large ρ . In this regime, however, the perturbative analysis based on the one loop beta function is not applicable.

2.3 Zero Modes and the $U(1)_A$ anomaly

In the last section we showed that instantons interpolate between different topological vacua in QCD. It is then natural to ask if the different vacua can be physically distinguished. This question is answered most easily in the presence of light fermions, because the different vacua have different axial charge. This observation is the key element in understanding the mechanism of chiral anomalies.

Anomalies first appeared in the context of perturbation theory [8,9]. From the triangle diagram involving an external axial vector current one finds that

² The question why θ happens to be so small is known as the “strong CP problem”.

Most likely, the resolution of the strong CP problem requires physics outside QCD and we will not discuss it any further.

the flavor singlet current which is conserved on the classical level develops an anomalous divergence on the quantum level

$$\partial_\mu j_\mu^5 = \frac{N_f}{16\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a. \quad (14)$$

This anomaly plays an important role in QCD, because it explains the absence of a ninth goldstone boson, the so called $U(1)_A$ puzzle.

The mechanism of the anomaly is intimately connected with instantons. First, we recognize the integral of the RHS of (14) as $2N_f Q$, where Q is the topological charge. This means that in the background field of an instanton we expect axial charge conservation to be violated by $2N_f$ units. The crucial property of instantons, originally discovered by 't Hooft, is that the Dirac operator has a zero mode $iD\psi_0(x) = 0$ in the instanton field. For an instanton in the singular gauge, the zero mode wave function is

$$\psi_0(x) = \frac{\rho}{\pi} \frac{1}{(x^2 + \rho^2)^{3/2}} \frac{\gamma \cdot x}{\sqrt{x^2}} \frac{1 + \gamma_5}{2} \phi \quad (15)$$

where $\phi^{\alpha m} = \epsilon^{\alpha m}/\sqrt{2}$ is a constant spinor, which couples the color index α to the spin index $m = 1, 2$. Note that the solution is left handed, $\gamma_5 \psi_0 = -\psi_0$. Analogously, in the field of an anti-instanton there is a right handed zero mode.

We can now see how axial charge is violated during tunneling. For this purpose, let us consider the Dirac Hamiltonian $i\boldsymbol{\alpha} \cdot \mathbf{D}$ in the field of the instanton. The presence of a 4-dimensional normalizable zero mode implies that there is one left handed state that crosses from positive to negative energy during the tunneling event. This can be seen as follows: In the adiabatic approximation, solutions of the Dirac equation are given by

$$\psi_i(\mathbf{x}, t) = \psi_i(\mathbf{x}, t = -\infty) \exp \left(- \int_{-\infty}^t dt' \epsilon(t') \right). \quad (16)$$

The only way we can have a 4-dimensional normalizable wave function is if ϵ_i is positive for $t \rightarrow \infty$ and negative for $t \rightarrow -\infty$. This explains how axial charge can be violated during tunneling. No fermion ever changes its chirality, all states simply move one level up or down. The axial charge comes, so to say, from the “bottom of the Dirac sea”.

2.4 The effective interaction between quarks

Proceeding from pure glue theory to QCD with light quarks, one has to deal with the much more complicated problem of quark-induced interactions. Indeed, on the level of a single instanton we can not even understand the presence of instantons in full QCD. The reason is again related to the existence of zero modes. In the presence of light quarks, the tunneling rate is proportional to the fermion determinant, which is given by the product of the eigenvalues

of the Dirac operator. This means that (as $m \rightarrow 0$) the tunneling amplitude vanishes and individual instantons cannot exist!

This result is related to the anomaly: During the tunneling event, the axial charge of the vacuum changes, so instantons have to be accompanied by fermions. The tunneling amplitude is non-zero only in the presence of external quark sources, because zero modes in the denominator of the quark propagator can cancel against zero modes in the determinant. Consider the fermion propagator in the instanton field

$$S(x, y) = \frac{\psi_0(x)\psi_0^+(y)}{im} + \sum_{\lambda \neq 0} \frac{\psi_\lambda(x)\psi_\lambda^+(y)}{\lambda + im} \quad (17)$$

where $iD\psi_\lambda = \lambda\psi_\lambda$. For N_f light quark flavors the instanton amplitude is proportional to m^{N_f} . Instead of the tunneling amplitude, let us calculate a $2N_f$ -quark Green's function $\langle \prod_f \bar{\psi}_f(x_f) \Gamma \psi_f(y_f) \rangle$, containing one quark and antiquark of each flavor. Performing the contractions, the amplitude involves N_f fermion propagators (17), so that the zero mode contribution involves a factor m^{N_f} in the denominator.

The result can be written in terms of an effective Lagrangian [11]. It is a non-local $2N_f$ -fermion interaction, where the quarks are emitted or absorbed in zero mode wave functions. The result simplifies if we take the long wavelength limit (in reality, the interaction is cut off at momenta $k > \rho^{-1}$) and average over the instanton position and color orientation. For $N_f = 1$ the result is [11,14]

$$\mathcal{L}_{N_f=1} = \int d\rho n_0(\rho) \left(m\rho - \frac{4}{3}\pi^2 \rho^3 \bar{q}_R q_L \right), \quad (18)$$

where $n_0(\rho)$ is the tunneling rate. Note that the zero mode contribution acts like a mass term. For $N_f = 1$, there is only one chiral $U(1)$ symmetry, which is anomalous. This means that the anomaly breaks chiral symmetry and gives a fermion mass term. This is not true for more than one flavor. For $N_f = 2$, the result is

$$\begin{aligned} \mathcal{L}_{N_f=2} = \int d\rho n_0(\rho) & \left[\prod_f \left(m\rho - \frac{4}{3}\pi^2 \rho^3 \bar{q}_{f,R} q_{f,L} \right) \right. \\ & \left. + \frac{3}{32} \left(\frac{4}{3}\pi^2 \rho^3 \right)^2 (\bar{u}_R \lambda^a u_L \bar{d}_R \lambda^a d_L - \bar{u}_R \sigma_{\mu\nu} \lambda^a u_L \bar{d}_R \sigma_{\mu\nu} \lambda^a d_L) \right]. \end{aligned} \quad (19)$$

One can easily check that the interaction is $SU(2) \times SU(2)$ invariant, but $U(1)_A$ is explicitly broken. This Lagrangian is of the type first studied by Nambu and Jona-Lasinio [12] and widely used as a model for chiral symmetry breaking and as an effective description for low energy chiral dynamics. It can be transformed to the form discussed above when we compare it to NJL interaction.

2.5 The quark condensate in the mean field approximation

We showed in the last section that in the presence of light fermions, tunneling can only take place if the tunneling event is accompanied by N_f fermions which change their chirality. But in the QCD vacuum, chiral symmetry is broken and the quark condensate $\langle\bar{q}q\rangle = \langle\bar{q}_L q_R + \bar{q}_R q_L\rangle$ is non-zero. This means that there is a finite amplitude for a quark to change its chirality and we expect the instanton density to be finite.

For a sufficiently dilute system of instantons, we can estimate the instanton density in full QCD from the expectation value of the $2N_f$ fermion operator in the effective Lagrangian (19). Using the factorization assumption [4], we find that the factor $\prod_f m_f$ in the instanton density should be replaced by $\prod_f m_f^*$, where the effective quark mass is given by

$$m_f^* = m_f - \frac{2}{3}\pi^2\rho^2\langle\bar{q}_f q_f\rangle. \quad (20)$$

This shows that if chiral symmetry is broken, the instanton density is finite in the chiral limit.

This obviously raises the question whether the quark condensate itself can be generated by instantons. This question can be addressed using several different techniques (for a review, see [1,2]). One possibility is to use the effective interaction (19) and to calculate the quark condensate in the mean field (Hartree-Fock) approximation. This corresponds to summing the contribution of all ‘‘cactus’’ diagrams to the full quark propagator. The result is a gap equation [13]

$$\int \frac{d^4k}{(2\pi)^4} \frac{M^2(k)}{k^2 + M^2(k)} = \frac{N}{4N_c V}, \quad (21)$$

which determines the constituent quark mass $M(0)$ in terms of the instanton density (N/V). Here, $M(k) = M(0)k^2\varphi'^2(k)/(2\pi\rho)$ is the momentum dependent effective quark mass and $\varphi'(k)$ is the Fourier transform of the zero mode profile [13]. The quark condensate is given by

$$\langle\bar{q}q\rangle = -4N_c \int \frac{d^4k}{(2\pi)^4} \frac{M(k)}{M^2(k) + k^2}. \quad (22)$$

Using our standard parameters $(N/V) = 1 \text{ fm}^{-4}$ and $\rho = 1/3 \text{ fm}$, one finds $\langle\bar{q}q\rangle \simeq -(255 \text{ MeV})^3$ and $M(0) = 320 \text{ MeV}$. Parametrically, $\langle\bar{q}q\rangle \sim (N/V)^{1/2}\rho^{-1}$ and $M(0) \sim (N/V)^{1/2}\rho$. Note that both quantities are not proportional to (N/V) , but to $(N/V)^{1/2}$. This is a reflection of the fact that spontaneous breaking of chiral symmetry is not a single instanton effect, but involves infinitely many instantons.

A very instructive way to study the mechanism for chiral symmetry breaking at a more microscopic level is by considering the distribution of eigenvalues of the Dirac operator. A general relations that connects the spectral

density $\rho(\lambda)$ of the Dirac operator to the quark condensate was given by Banks-Casher relation

$$\langle \bar{q}q \rangle = -\pi\rho(0). \quad (23)$$

This result is analogous to the Kondo formula for the electrical conductivity. Just like the conductivity is given by the density of states at the Fermi surface, the quark condensate is determined by the level density at zero virtuality λ . For a disordered, random, system of instantons the zero modes interact and form a band around $\lambda = 0$. As a result, the eigenstates are delocalized and chiral symmetry is broken. On the other hand, if instantons are strongly correlated, for example bound into topologically neutral molecules, the eigenvalues are pushed away from zero, the eigenstates are localized and chiral symmetry is unbroken. As we will see below, precisely which scenario is realized depends on the parameters of the theory, like the number of light flavors and the temperature. Of course, for “real” QCD with two light flavors at $T = 0$, we expect chiral symmetry to be broken. This is supported by numerical simulations of the partition function of the instanton liquid, see [1].

2.6 The Qualitative Picture of the Instanton Ensemble

Using basically such expressions and the known value of the quark condensate it was pointed out in[5] that all would be consistent only if the typical instanton size happened to be significantly smaller than their separation³, $R = n^{-1/4} \approx 1\text{ fm}$, namely $\rho_{\max} \sim 1/3\text{ fm}$.

In Fig.(1) one can see lattice data on instanton size distribution, obtain by cooling of the original gauge fields. Similar distribution can also be obtained from fermionic lowest Dirac eigenmodes: in this case no “cooling” is needed.

Let me now show another evidence for this value of the instanton size, taken from the pion form-factor calculated[15] in the instanton model. In Fig.(2) we show how the experimentally measured pion size correlates with the input mean instanton size: one can see from it that the value .35 fm is a clear winner.

If so, the following qualitative picture of the QCD vacuum have emerged:

1. Since the instanton size is significantly smaller than the typical separation R between instantons, $\rho/R \sim 1/3$, the vacuum is fairly dilute. The fraction of spacetime occupied by strong fields is only a few percent.
2. The fields inside the instanton are very strong, $G_{\mu\nu} \gg \Lambda_{QCD}^2$. This means that the semi-classical approximation is valid, and the typical action is large

$$S_0 = 8\pi^2/g^2(\rho) \sim 10 - 15 \gg 1. \quad (24)$$

Higher order corrections are proportional to $1/S_0$ and presumably small.

³ Derived in turn from the gluon condensate and the topological susceptibility.

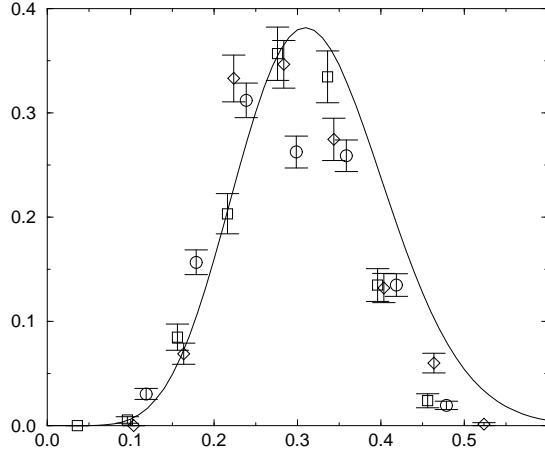


Fig. 1. The instanton density $dn/d\rho d^4z$, [fm^{-5}] versus its size ρ [fm]. The points are from the lattice work [10], for this theory, with $\beta=5.85$ (diamonds), 6.0 (squares) and 6.1 (circles). Their comparison should demonstrate that results are rather lattice-independent. The line corresponds to the proposed expression $\sim \exp(-2\pi\sigma\rho^2)$, see text.

3. Instantons retain their individuality and are not destroyed by interactions. From the dipole formula, one can estimate

$$|\delta S_{int}| \sim (2 - 3) \ll S_0. \quad (25)$$

4. Nevertheless, interactions are important for the structure of the instanton ensemble, since

$$\exp |\delta S_{int}| \sim 20 \gg 1. \quad (26)$$

This implies that interactions have a significant effect on correlations among instantons, and the instanton ensemble in QCD is not a dilute gas but an *interacting liquid*.

The aspects of the QCD vacuum for which instantons are most important are those related to light fermions. Their importance in the context of chiral symmetry breaking is related to the fact that the Dirac operator has a chiral zero mode in the field of an instanton. These zero modes are localized quark states around instantons, like atomic states of electrons around nuclei. At finite density of instantons those states can become collective, like atomic states in metals. The resulting de-localized state corresponds to the wave function of the quark condensate.

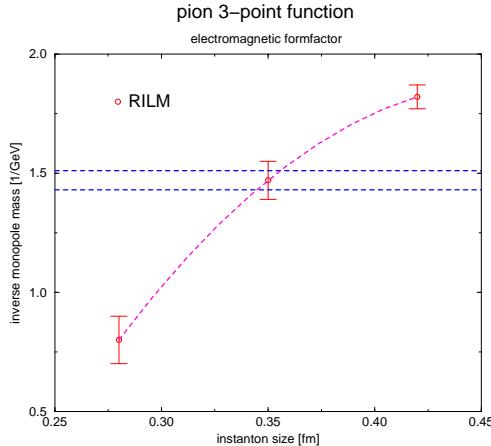


Fig. 2. The fitted parameter M of the pion form-factor $ff \sim M^2/(Q^2 + M^2)$ versus the inputted instanton size.

Direct tests of all these ideas on the lattice are possible. One may have a look at the lowest eigenmodes and see if they are related to instantons or something else (monopoles, vortices...) by identifying their shapes - 4d bumps (lines or 2-d sheets) respectively. So far, only bumps (that is the instantons) were seen.

One may also test how locally chiral are the lowest eigenmodes. Just at this school I learned from one of its young participants, Christof Gattringer, about his version of chiral lattice fermions and nice results he got. Among those is remarkably well defined separation between instanton-based and perturbative-like lowest modes, revealed by the so called participation ratios.

Let me now explain about the *lowest QCD scale* generated by instantons, mentioned above. The width of the *zero mode zone* of states is of the order of root-mean-square matrix element of the Dirac operator $\langle I|D|J \rangle \sim \rho^2/R^3$. Here states I, J are some instanton and anti-instanton zero modes, ρ is the instanton size and $R \sim n^{-1/4} \approx 1\text{ fm}$ is the distance between their centers. Note small factor $(\rho/R)^2 \sim 1/10$ here. The Dirac eigenvalues from the zone have similar magnitude. Now, the eigenvalues enter together with quark mass m : and so only when this quark mass is smaller than this scale we start seeing the physics of the zero mode zone. In particular, for *quenched QCD* (or instanton liquid) there is no determinant and the zone states have rather wrong spectrum. However, only if the quark mass is small compared to its width we start observing the difference. Only recently lattice practitioners were able to do so: indeed, quenched QCD results at small m start deviating from the correct answers quite drastically.

2.7 Interacting instantons

In the QCD partition function there are two types of fields, gluons and quarks, and so the first question one addresses is *which integral to take first*.

(i) One way is to eliminate *gluonic* degrees of freedom first. Physical motivation for this may be that gluonic states are heavy and an effective fermionic theory should be better suited to derive an effective low-energy fermionic theory. It is a well-trodden path and one can follow it to the development of a similar four-fermion theory, the NJL model. One can do simple mean field or random field approximation (RPA) diagrams, and find the mean condensate and properties of the Goldstone mesons[13]. The results for Color Superconductors at high density reported below are done with the same technique as well. But nevertheless, not much can really be done in such NJL-like approach. In fact, multiple attacks during the last 40 years at the NJL model *beyond the mean field* basically failed. In particular, one might think that since baryons are states with three quarks, and one may wonder if using quasi-local four-fermion Lagrangians for the three body problem is a solvable quantum mechanical problem, and one can at least tell if nucleons are or are not bound in NJL. In fact it is not: the results depend strongly on subtleties of how the local limit for the interaction is defined, and there is no clear answer to this question. Other notorious attempts to sum more complicated diagrams deal with the possible modification of the the chiral condensate. Some works even claim that those diagrams destroy it *completely!*

Going from NJL to instantons improves the situation enormously: the shape of the form-factor is no longer a guess (it is provided by the shape of zero modes) and one can in principle evaluate any particular diagram. However *summing them all up* still seems like an impossible task.

(ii) The solution to this problem was found. For that one has to follow the opposite strategy and do the *fermion* integral first. The first step is simple and standard: fermions only enter quadratically, leading to a fermionic determinant. In the instanton approximation, it leads to the Interacting Instanton Liquid Model, defined by the following partition function:

$$Z = \sum_{N_+, N_-} \frac{1}{N_+! N_-!} \int \prod_i^{N_+ + N_-} [d\Omega_i d(\rho_i)] \exp(-S_{\text{int}}) \prod_f^{N_f} \det(\hat{D} + m_f) \quad (27)$$

describing a system of pseudo-particles interacting via the bosonic action and the fermionic determinant. Here $d\Omega_i = dU_i d^4 z_i d\rho_i$ is the measure in color orientation, position and size associated with single instantons, and $d(\rho)$ is the single instanton density $d(\rho) = dn_{I,\bar{I}}/d\rho dz$.

The gauge interaction between instantons is approximated by a sum of pure two-body interaction $S_{\text{int}} = \frac{1}{2} \sum_{I \neq J} S_{\text{int}}(\Omega_{IJ})$. Genuine three body effects in the instanton interaction are not important as long as the ensemble is reasonably dilute. Implementation of this part of the interaction (quenched simulation) is quite analogous to usual statistical ensembles made of atoms.

As already mentioned, quark exchanges between instantons are included in the fermionic determinant. Finding a diagonal set of fermionic eigenstates of the Dirac operator is similar to what people are doing, e.g., in quantum chemistry when electron states for molecules are calculated. The difficulty of our problem is however much higher, because this set of fermionic states should be determined for *all* configurations which appear during the Monte-Carlo process.

If the set of fermionic states is however limited to the subspace of instanton zero modes, the problem becomes tractable numerically. Typical calculations in the IILM involved up to $N \sim 100$ instantons (+anti-instantons): which means that the determinants of $N \times N$ matrices are involved. Such determinants can be evaluated by an ordinary workstation (and even PC these days) so quickly that a straightforward Monte Carlo simulation of the IILM is possible in a matter of minutes. On the other hand, expanding the determinant in a sum of products of matrix elements, one can easily identify the sum of all closed loop diagrams up to order N in the 't Hooft interaction. Thus, in this way one can actually take care of about 100 factorial diagrams!

3 Hadronic Structure and the QCD correlation functions.

3.1 Correlators as a bridge between hadronic and partonic worlds

Consider two currents separated by a *space-like* distance x (which can be considered as the spatial distance, or an Euclidean time) and introduce correlation functions of the type

$$K(x) = \langle T(J(x)J(0)) \rangle \quad (28)$$

with $J(x) = \bar{\psi}(x)\Gamma\psi(x)$. The matrix Γ contains γ_μ for vector currents, γ_5 for the pseudoscalar or 1 for the scalars, etc, and also a flavor matrix, if needed.

We will start with isovector vector and axial currents, and then discuss 4 scalar-pseudoscalar channels: π ($P=-1, I=1$), σ or f_0 ($P=+1, I=0$), η ($P=-1, I=0$) and δ or a_0 ($P=+1, I=1$).

In a (relativistic) field theory, correlation functions of gauge invariant local operators are the proper tool to study the spectrum of the theory. The correlation functions can be calculated either from the physical states (mesons, baryons, glueballs) or in terms of the fundamental fields (quarks and gluons) of the theory. In the latter case, we have a variety of techniques at our disposal, ranging from perturbative QCD, the operator product expansion (OPE), to models of QCD and lattice simulations. For this reason, correlation functions provide a bridge between hadronic phenomenology on the one side and the underlying structure of the QCD vacuum on the other side.

Loosely speaking, hadronic correlation functions play the same role for understanding the forces between quarks as the NN scattering phase shifts

did in the case of nuclear forces. In the case of quarks, however, confinement implies that we cannot define scattering amplitudes in the usual way. Instead, one has to focus on the behavior of gauge invariant correlation functions at short and intermediate distance scales. The available theoretical and phenomenological information about these functions was recently reviewed in [3].

In all cases at small x we expect $K(x) \approx K_0(x)$ where the latter corresponds to just *free* propagation of (about massless) light quarks. The zeroth order correlators are all just $K_0(x) = 12/(\pi^4 x^6)$, basically the square of the massless quark propagator.

The first deviations due to non-perturbative effects can be studied using Wilsonian Operator Product Expansion (OPE) in ref[4]. For all scalar and pseudoscalar channels the resulting first correction is

$$\frac{K(x)}{K_0(x)} = 1 + \frac{x^4}{384} <(gG)^2> + \dots \quad (29)$$

The “gluon condensate” is assumed to be made out of a soft vacuum field, and therefore all arguments can be simply taken at the point $x = 0$. The so-called *standard* value of the “gluon condensate” appearing here was estimated previously from charmonium sum rules:

$$<(gG)^2>_{SVZ} \approx .5 \text{ GeV}^4 \quad (30)$$

Thus, the OPE suggests the following scale, at which the correction becomes equal to the first term:

$$x_{OPE} = (384 / <(gG)^2>_{SVZ})^{1/4} \approx 1.0 \text{ fm} \quad (31)$$

This seems to be completely consistent with the approximation used. However, as Novikov, Shifman, Vainshtein and Zakharov soon noticed[6], this (and other OPE corrections) completely failed to describe all the $J^P = O^\pm$ channels: we return to this issue after we consider vectors and axialis.

3.2 Vector and axial correlators

The information available on vector correlation functions from experimental data on $e^+e^- \rightarrow \text{hadrons}$, the OPE and other exact results was reviewed in [3]. Since then, however, new high statistics measurement of hadronic τ decays $\tau \rightarrow \nu_\tau + \text{hadrons}$ have been done. For definiteness, we use results of one of them, ALEPH experiment at CERN [16,17].

The vector and axial-vector correlation functions are $\Pi_V(x) = \langle j_\mu^a(x) j_\mu^a(0) \rangle$ and $\Pi_A(x) = \langle j_\mu^{5a}(x) j_\mu^{5a}(0) \rangle$. Here, $j_\mu^a(x) = \bar{q} \gamma_\mu \frac{\tau^a}{2} q$, $j_\mu^{5a}(x) = \bar{q} \gamma_\mu \gamma_5 \frac{\tau^a}{2} q$ are the isotriplet vector and axial-vector currents. The Euclidean correlation functions have the spectral representation [3]

$$\Pi_{V,A}(x) = \int ds \rho_{V,A}(s) D(\sqrt{s}, x), \quad (32)$$

where $D(m, x) = m/(4\pi^2 x)K_1(mx)$ is the Euclidean coordinate space propagator of a scalar particle with mass m . We shall focus on the linear combinations $\Pi_V + \Pi_A$ and $\Pi_V - \Pi_A$. These combinations allows for a clearer separation of different non-perturbative effects. The corresponding spectral functions $\rho_V \pm \rho_A$ measured by the ALEPH collaboration are shown in Fig. 3. The errors are a combination of statistical and systematic ones (below we use them conservatively, as pure systematic): the main problem seems to be separation into V and A of channels with Kaons, which may affect $V - A$ at $s > 2 \text{ GeV}$ at 10% level. None of our conclusions are sensitive to it.

In QCD, the vector and axial-vector spectral functions must satisfy chiral sum rules. Assuming that $\rho_V - \rho_A = 0$ at above $s > m_\tau^2$, and using ALEPH data below it, one finds that all 4 of the sum rules are satisfied within the experimental uncertainty, but the central values differ significantly from the chiral predictions [16]. In general, both functions are expected to have oscillations of decreasing amplitude, and putting $\rho_V - \rho_A$ to zero at arbitrary point imply appearance of spurious dimension $d = 2, 4$ operators in the correlation functions at small x . Therefore, we have decided to terminate the data above a specially tuned point, $s_0 = 2.5 \text{ GeV}^2$, enforcing all 4 chiral sum rules. (The reader should however be aware of the fact that we have, in effect, slightly moved the data points in the small x region within the error band.) Finally we add the pion pole contribution (not shown in Fig. 3), which corresponds to an extra term $\Pi_A^\pi(x) = f_\pi^2 m_\pi^2 D(m_\pi, x)$. The resulting correlation functions $\Pi_V(x) \pm \Pi_A(x)$ are shown in Figs. 4.

We begin our analysis with the combination $\Pi_V - \Pi_A$. This combination is sensitive to chiral symmetry breaking, while perturbative diagrams, as well as gluonic operators cancel out.

In Fig. 4 we compare the measured correlation functions with predictions from the instanton liquid model (in its simplest form, random instanton liquid with parameters n, ρ fixed in [5] and discussed above).

The agreement of the instanton prediction with the measured $V - A$ correlation is impressive: it extends all the way from short to large distances. At distances $x > 1.25 \text{ fm}$ both combinations are dominated by the pion contribution while at intermediate x the ρ, ρ' and a_1 resonances contribute.

We shall now focus our attention on the $V + A$ correlation function. The unique feature of this function is the fact that the correlator remains close to free field behavior for distances as large as 1 fm. This phenomenon was referred to as “super-duality” in [3]. The instanton model reproduces this feature of the $V + A$ correlator. We also notice that for small x the deviation of the correlator in the instanton model from free field behavior is small compared to the perturbative $O(\alpha_2/\pi)$ correction. This opens the possibility of precision studies of the pQCD contribution. But before we do so, let us compare the correlation functions to the OPE prediction

$$\frac{\Pi_V(x) + \Pi_A(x)}{2\Pi_0(x)} = 1 + \frac{\alpha_s}{\pi} - \frac{1}{384} \langle g^2 G_{\mu\nu}^2 \rangle x^4$$

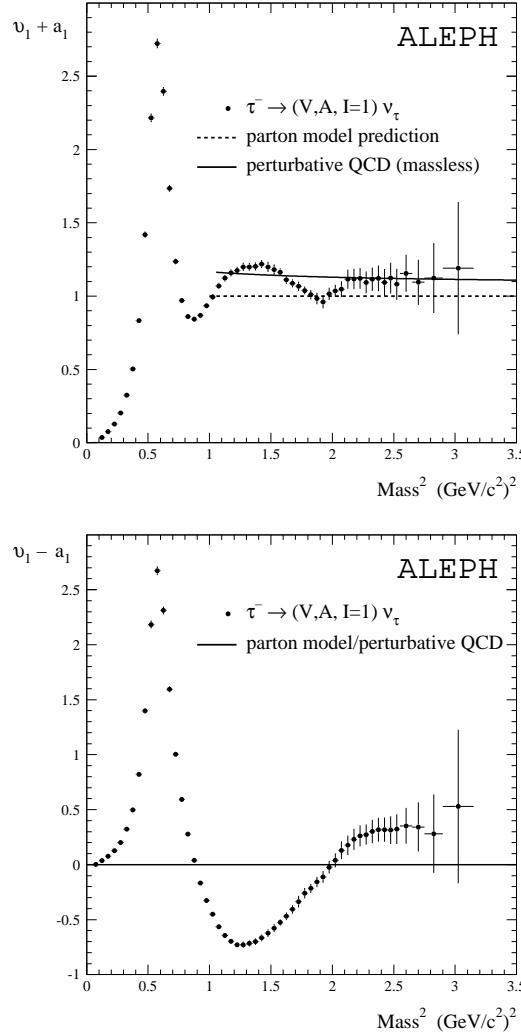


Fig. 3. Spectral functions $v(s) \pm a(s) = 4\pi^2(\rho_V(s) + \rho_A(s))$ extracted by the ALEPH collaboration from τ lepton hadronic decays.

$$-\frac{4\pi^3}{81}\alpha_s(x)\langle\bar{q}q\rangle\log(x^2)x^6 + \dots \quad (33)$$

Note that the perturbative correction is attractive, while the power corrections of dimension $d = 4$ and $d = 6$ are repulsive. Direct instantons also induce an $O(x^4)$ correction $1 - \frac{\pi^2}{12} \left(\frac{N}{V}\right) x^4 + \dots$, which is consistent with the OPE because in a dilute instanton liquid we have $\langle g^2 G^2 \rangle = 32\pi^2(N/V)$. This

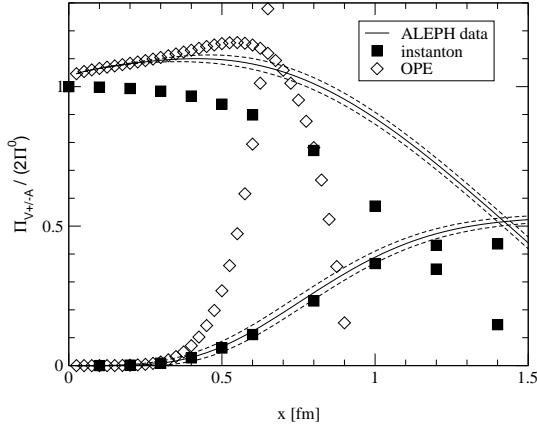


Fig. 4. Euclidean coordinate space correlation functions $\Pi_V(x) \pm \Pi_A(x)$ normalized to free field behavior. The solid lines show the correlation functions reconstructed from the ALEPH spectral functions and the dotted lines are the corresponding error band. The squares show the result of a random instanton liquid model and the diamonds the OPE fit described in the text.

term can indeed be seen in the instanton calculation and causes the correlator to drop below 1 at small x . It is possible to extract the value of Λ_{QCD} (we find $\alpha_s(m_\tau) = 0.35$) and even clear indication of running coupling. It is only possible to do because the non-perturbative corrections (represented by instantons) are basically cancelling each other to very high degree, in $V+A$ channel.

Why is it happening? The first order in 't Hooft is indeed absent, due to chirality mismatch. There is no general theoretical reason why all non-perturbative of higher order should also do so: but ALEPH data used wrongly hint that they actually do so.

3.3 Spin-zero correlation functions

Now we will see case4s which are completely opposite to those just considered: the instanton-induced effects would be large. Furthermore, the 4 channels actually show completely different non-perturbative deviation from K_0 at small x : half of them (π, σ) deviate upward, and another pair (η, δ) deviate downward.

But let me first demonstrate that the OPE scale determined above cannot be right. All we have to do is to evaluate the strength of the pion contribution

to the correlator in question:

$$K_\pi(x) = \frac{\lambda_\pi^2}{4\pi^2 x^2} \quad (34)$$

The coupling constant is defined as $\lambda_\pi = <0|J(0)|\pi>$ and the rest is nothing more than the scalar massless propagator⁴. Because both the pion term and the gluon condensate correction happen to be $1/x^2$, let us compare the coefficients. Ideal matching would mean they are about the same

$$\lambda_\pi^2 \approx \frac{<(gG)^2>_{SVZ}}{8\pi^2} \quad (35)$$

The r.h.s. is about 0.0063 GeV⁴. However, phenomenology tells us that (unlike the better known coupling to the axial current f_π) the coupling λ_π is surprisingly large⁵. The l.h.s. of this relation is actually $\lambda_\pi^2 = (.48 \text{ GeV})^4 = 0.053 \text{ GeV}^4$, about *10 times larger* than the r.h.s. It means much larger non-perturbative effect is needed to explain the deviation from the perturbative behavior.

Now, let us see why is it so. The instanton effects in spin-0 channels are in these cases much larger because effect of 't Hooft interaction appears in those cases in the first order. Furthermore, since its flavor structure is non-diagonal ($\bar{u}u$)($\bar{d}d$) the correlator of two π^0 currents ($\bar{u}\gamma_5 u - \bar{d}\gamma_5 d$) have it with opposite sign as compared to the correlator of η' currents ($\bar{u}\gamma_5 u + \bar{d}\gamma_5 d$). What it means is that instantons are as attractive in the pion channel as they are repulsive in the η' case. The situation is reversed in the scalar channels: the isoscalar sigma is attractive and isovector is repulsive.

Full results from versions of the instanton liquid model for pion correlators are shown in fig.5. Different versions of the model (mentioned in figures below as IILM(rat) etc) differ by a particular ansatz for the gauge field used, from which the interaction is calculated. Note also, that these figures contain also a curve marked "phen": this is what the correlator actually looks like, according to phenomenology.

We simply show a few results of correlation functions in the different instanton ensembles (see original refs in[1]). Some of them (like vector and axial-vector ones) turned out to be easy: nearly any variant of the instanton model can reproduce the (experimentally known!) correlators well. Some of them are sensitive to details of the model very much: two such cases are shown in Figs. 5-6. The pion correlation functions in the different ensembles are qualitatively very similar. The differences are mostly due to different values of the quark condensate (and the physical quark mass) in the different

⁴ We can ignore the pion mass at the distances in question. We also ignore contributions of other states, which can only add positively to the correlator and made disagreement only worse.

⁵ The reason for that is the the pion is rather compact and also the wave function is concentrated at its center, so that its value at $r = 0$ is large. We return to this point in the discussion of the "instanton liquid" model.

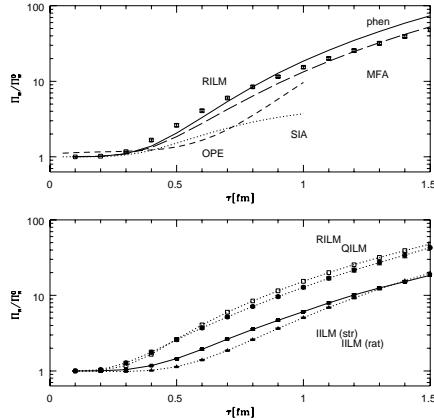


Fig. 5. Pion correlation function in various approximations and instanton ensembles. In the top figure we show the phenomenological expectation (solid), the OPE (dashed), the single instanton (dash-dotted) and mean field approximations (dashed) as well as data in the random instanton ensemble. In the bottom figure we compare different instanton ensembles, random (open squares), quenched (circles) and interacting (streamline: solid squares, ratio ansatz solid triangles).

ensembles. Using the Gell-Mann-Oaks-Renner relation, one can extrapolate the pion mass to the physical value of the quark masses. The results are consistent with the experimental value in the streamline ensemble (both quenched and unquenched), but clearly too small in the ratio ansatz ensemble. This is a reflection of the fact that the ratio ansatz ensemble is not sufficiently dilute.

The situation is drastically different in the η' channel. Among the ~ 40 correlation functions calculated in the random ensemble, only the η' and the isovector-scalar δ were found to be completely unacceptable. The correlation function decreases very rapidly and becomes *negative* at $x \sim 0.4$ fm. This behavior is incompatible even with a normal spectral representation. The interaction in the random ensemble is too repulsive, and the model “over-explains” the $U(1)_A$ anomaly.

The results in the unquenched ensembles (closed and open points) significantly improve the situation. This is related to dynamical correlations between instantons and anti-instantons (topological charge screening). The single instanton contribution is repulsive, but the contribution from pairs is attractive. Only if correlations among instantons and anti-instantons are sufficiently strong are the correlators prevented from becoming negative. Quantitatively, the δ and η_{ns} masses in the streamline ensemble are still too heavy as compared to their experimental values. In the ratio ansatz, on the other hand, the correlation functions even show an enhancement at distances on the order of 1 fm, and the fitted masses are too light. This shows that the η' channel is very sensitive to the strength of correlations among instantons.

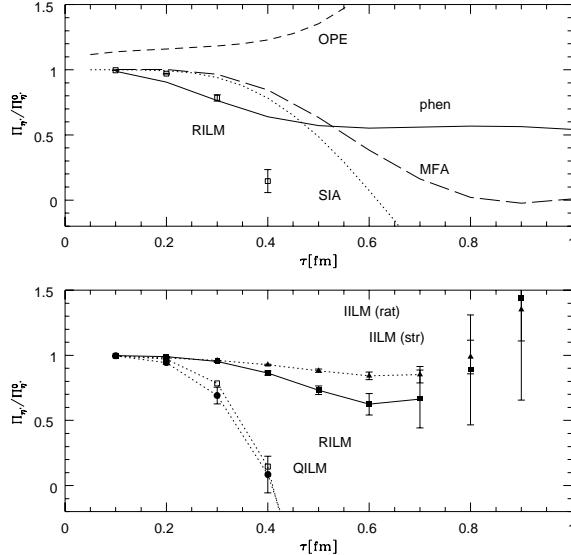


Fig. 6. Eta prime meson correlation functions. The various curves and data sets are labeled as in Fig. 5. Note that random instanton liquid model (RILM) and quenched version (no fermionic determinant, only bosonic interactions) predict η' correlator to go negative. The same unphysical behavior has been found on the lattice.

In summary, pion properties are mostly sensitive to global properties of the instanton ensemble, in particular its diluteness. Good phenomenology demands $\bar{\rho}^4 n \simeq 0.03$, as originally suggested in [5]. The properties of the ρ meson are essentially independent of the diluteness, but show sensitivity to $\bar{I}I$ correlations. These correlations become crucial in the η' channel.

3.4 Baryonic correlation functions

The existence of a strongly attractive interaction in the pseudoscalar quark-antiquark (pion) channel also implies an attractive interaction in the scalar quark-quark (diquark) channel. This interaction is phenomenologically very desirable, because it immediately explains why the nucleon is light, while the delta ($S=3/2, I=3/2$) is heavy.

The so called Ioffe currents (with no derivatives and the minimum number of quark fields) are local operators which can excite states with nucleon quantum numbers. Those with positive parity and spin 1/2 can also be represented in terms of scalar and pseudoscalar diquarks

$$\eta_{1,2} = (2, 4) \left\{ \epsilon_{abc} (u^a C d^b) \gamma_5 u^c \mp \epsilon_{abc} (u^a C \gamma_5 d^b) u^c \right\}. \quad (36)$$

Nucleon correlation functions are defined by $\Pi_{\alpha\beta}^N(x) = \langle \eta_\alpha(0) \bar{\eta}_\beta(x) \rangle$, where α, β are the Dirac indices of the nucleon currents. In total, there are six

different nucleon correlators: the diagonal $\eta_1\bar{\eta}_1$, $\eta_2\bar{\eta}_2$ and off-diagonal $\eta_1\bar{\eta}_2$ correlators, each contracted with either the identity or $\gamma \cdot x$. Let us focus on the first two of these correlation functions (for more detail, see[1] and references therein).

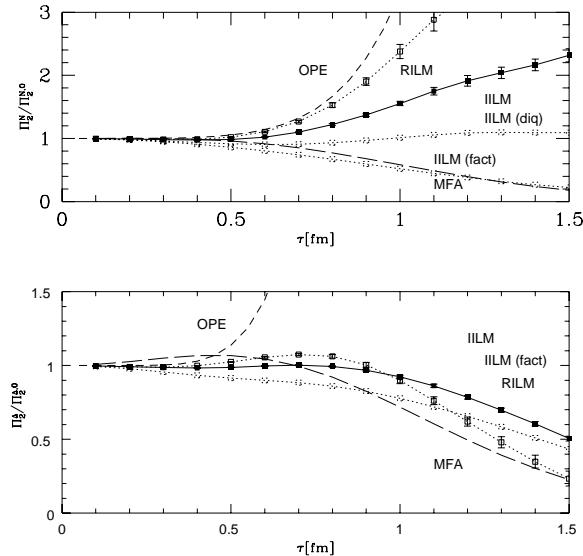


Fig. 7. Nucleon and delta correlation functions Π_2^N and Π_2^Δ . Curves labeled as in Figs.on mesonic correlators.

The correlation function Π_2^N in the interacting ensemble is shown in Fig. 7. The fact that the nucleon in IILM is actually bound can also be demonstrated by comparing the full nucleon correlation function with that of three non-interacting quarks (the cube of the average propagator). The full correlator is significantly larger than the non-interacting one.

There is a significant enhancement over the perturbative contribution which is nicely described in terms of the nucleon contribution. Numerically, we find⁶ $m_N = 1.019$ GeV. In the random ensemble, we have measured the nucleon mass at smaller quark masses and found $m_N = 0.96 \pm 0.03$ GeV. The nucleon mass is fairly insensitive to the instanton ensemble. However, the strength of the correlation function depends on the instanton ensemble. This is reflected by the value of the nucleon coupling constant, which is smaller in the IILM. In[18] we studied all six nucleon correlation functions. We showed that all correlation functions can be described with the same nucleon mass and coupling constants.

⁶ Note that this value corresponds to a relatively large current quark mass $m = 30$ MeV.

The fitted value of the threshold is $E_0 \simeq 1.8$ GeV, indicating that there is little strength in the “three quark continuum” (dual to higher resonances in the nucleon channel). A significant part of this interaction was traced down to the strongly attractive *scalar diquark* channel. The nucleon (at least in IILM) is a strongly bound diquark, plus a loosely bound third quark. The properties of this diquark picture of the nucleon continue to be disputed by phenomenologists. We will return to diquarks in the next section, where they will become Cooper pairs of Color Super-conductors.

In the case of the Δ resonance, there exists only one independent Ioffe current, given (for the Δ^{++}) by

$$\eta_\mu^\Delta = \epsilon_{abc}(u^a C \gamma_\mu u^b) u^c. \quad (37)$$

However, the spin structure of the correlator $\Pi_{\mu\nu;\alpha\beta}^\Delta(x) = \langle \eta_{\mu\alpha}^\Delta(0) \bar{\eta}_{\nu\beta}^\Delta(x) \rangle$ is much richer. In general, there are ten independent tensor structures, but the Rarita-Schwinger constraint $\gamma^\mu \eta_\mu^\Delta = 0$ reduces this number to four.

The mass of the delta resonance is too large in the random model, but closer to experiment in the unquenched ensemble. Note that, similar to the nucleon, part of this discrepancy is due to the value of the current mass. Nevertheless, the delta-nucleon mass splitting in the unquenched ensemble is $m_\Delta - m_N = 409$ MeV, larger but comparable to the experimental value 297 MeV. It mostly comes from the *absent scalar diquarks* in Δ channel.

4 The Phases of QCD

4.1 The Phase Diagram

In this section we discuss QCD in extreme conditions, such as finite temperature/density. Let me first emphasize why it is interesting and instructive to do. It is not simply to practice once again the semi-classical or perturbative methods similar to what have been done before in vacuum. What we are looking for here are *new phases* of QCD (and related theories), namely new self-consistent solutions which differ qualitatively from what we have in the QCD vacuum.

One such phase occurs at high enough temperature $T > T_c$: it is known as Quark Gluon Plasma (QGP). It is a phase understandable in terms of basic quark and gluon-like excitations[40], without confinement and with unbroken chiral symmetry in the massless limit⁷. One of the main goals of heavy ion program, especially at new the dedicated Brookhaven facility RHIC, is to study transitions to this phase.

Another one, which has been getting much attention recently, is the direction of finite density. Very robust Color Superconductivity was found to

⁷ It does not mean though, that it is a simple issue to understand even the high-T limit of QCD, related to non-perturbative 3d dynamics.

be the case here. Let me also mention one more frontier which has not yet attracted sufficient attention: namely a transition (or many transitions?) as the number of light flavors N_f grows. The minimal scenario includes a transition from the usual hadronic phase to a more unusual QCD phase, the *conformal* one, in which there are no particle-like excitations and correlators are power-like in the infrared. Even the position of the critical point is unknown. The main driving force of these studies is the intellectual challenge it provides.

The QCD phase diagram as we understand it now is shown in Fig 8(a), in the baryonic chemical potential μ (normalized per quark, not per baryon) and the temperature T plane. Some part of it is old: it has the hadronic phase at small values of both parameters, and QGP phase at large T, μ .

The phase transition line separating them most probably does not really start at $T = T_c, \mu = 0$ but at an “endpoint” E, a remnant of the so called QCD tricritical point which QCD has in the chiral (all quarks are massless) limit. Although we do not know where it is⁸, we hope to find it one day in experiment. The proposed ideas rotate around the fact that the order parameter, the VEV of the sigma meson, is at this point truly massless, and creates a kind of “critical opalescence”. Similar phenomena were predicted and then indeed observed at the endpoint of another line (called M from multi-fragmentation), separating liquid nuclear matter from the nuclear gas phase.

The large-density (and low-T) region looks rather different from what was shown at conferences just a year ago: two new Color Super-conducting phases appear there. Unfortunately heavy ion collisions do not cross this part of the phase diagrams and so it belongs to neutron star physics.

Above I mentioned an approach to high density starting from the vacuum. One can also work out in the opposite direction, starting from very large densities and going down. Since the electric part of one-gluon exchange is screened, and therefore the Cooper pairs appear due to magnetic forces. It is interesting by itself, as a rare example: one has to take care of *time delay effects* of the interaction. The result is indefinitely growing gaps at large $\mu > 10\text{GeV}$, as [36] $\Delta \sim \mu \exp(-\frac{3\pi^2}{\sqrt{2}g(\mu)})$.

4.2 Finite Temperature transition and Large Number of Flavors

There is no place here to discuss in detail the rather extensive lattice data available now, and I only mention some results related to instantons. In the vacuum a quasi-random set of instantons leads to chiral symmetry breaking and quasi-zero modes: but what in the same terms does the high-T phase look like?

The simplest solution would be just *suppression* of instantons at $T > T_c$, and at some early time people thought this is what actually happens. However, it should not be like this because the Debye screening which is

⁸ Its position is very sensitive to the precise value of the strange quark mass m_s

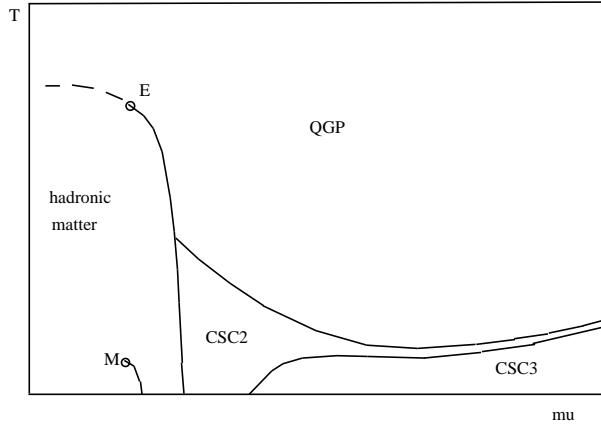


Fig. 8. Schematic phase diagram of QCD, in temperature T - baryon chemical potential μ plane. E and M show critical endpoints of first order transitions: M (from multi-fragmentation) is that for liquid-gas transition in nuclear matter. The color superconducting phases, CSC2 and CSC3 are explained in the text.

killing them only appears at $T = T_c$. Lattice data works have also found no depletion of the instanton density up to $T = T_c$.

On the other hand, the absence of the condensate and quasi-zero modes implies that the “liquid” is now broken into finite pieces. The simplest of them are pairs, or the instanton-anti-instanton molecules. This is precisely what instanton simulations have found[1], see fig.9. Whether it is indeed so on the lattice is not yet clear: nice molecules were located, but the evidence for the molecular mechanism of chiral restoration is still far from being convincing. (No alternative I am aware of have been so far proposed, though.)

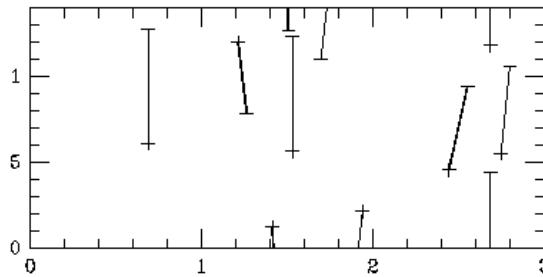


Fig. 9. Typical configuration from instanton liquid simulation, at $T > T_c$. Lines indicate the direction in which quark propagators are the largest. Clear pairing of instantons and instantons are observed: the pairs tend to have the same spatial position and being separated mostly by Euclidean time.

The results of IILM simulations with variable number of flavors $N_f = 2, 3, 5^9$ flavors with equal masses can be summarized as follows. For $N_f = 2$ there is a second order phase transition which turns into a line of first order transitions in the $m - T$ plane for $N_f > 2$. If the system is in the chirally restored phase ($T > T_c$) at $m = 0$, we find a discontinuity in the chiral order parameter if the mass is increased beyond some critical value. Qualitatively, the reason for this behavior is clear. While increasing the temperature increases the role of correlations caused by fermion determinant, increasing the quark mass has the opposite effect. We also observe that increasing the number of flavors lowers the transition temperature. Again, increasing the number of flavors means that the determinant is raised to a higher power, so fermion induced correlations become stronger. For $N_f = 5$ we find that the transition temperature drops to zero and the instanton liquid has a chirally symmetric ground state, provided the dynamical quark mass is less than some critical value. Studying the instanton ensemble in more detail shows that in this case, all instantons are bound into molecules.

Unfortunately, little is known about QCD with large numbers of flavors from lattice simulations. There are data by the Columbia group for $N_f = 4$. The most important result is that chiral symmetry breaking effects were found to be drastically smaller as compared to $N_f = 0, 2$. In particular, the mass splittings between chiral partners such as $\pi - \sigma$, $\rho - a_1$, $N(\frac{1}{2}^+) - N(\frac{1}{2}^-)$, extrapolated to $m = 0$ were found to be 4-5 times smaller. This agrees well with what was found in the interacting instanton model: more work in this direction is certainly needed.

4.3 High Density and Color Superconductivity

This topic is covered in detail by Prof. Alford at this school, so I only add few remarks to his lectures.

Although the idea of color superconductivity originates from 70's, the field of high density QCD was in the dormant state for long time till two papers [21,22] (posted on the same day) in 1998 have claimed gaps about 100 times larger than previously thought. The field is booming since, as one can see from about 250 citations in 2 years those papers got.

Then-Princeton group (Alford-Rajagopal-Wilczek) have been thinking about different pairings from theory perspective, but our (Stony Brook) team (Rapp,Schafer,ES,Velkovsky) had started from the impressive qq pairing phenomenon found theoretically [18] in the instanton liquid model *inside the nucleon*. As explained above, we have found it to be, roughly speaking, a small drop of CS matter, made of one Cooper pair of sort (the *ud scalar diquark*) and one massive quark¹⁰. T.Schafer heroically attempted numer-

⁹ The case $N_f = 4$ is omitted because in this case it is very hard to determine whether the phase transition happens at $T > 0$.

¹⁰ As opposed to Δ (decuplet) baryons, which is a small drop of "normal" quark matter, without scalar diquarks.

ical simulations of the instanton liquid model at finite μ : although he was not very successful¹¹ he found out strange “polymers” made of instantons connecting by 2 through going quark lines. It take us some time to realize we see paths of condensed diquarks! It was like finding superconductivity by watching electrons moving on your computer screen.

The main point I would like to emphasize here is that the qq pairing of such diquarks have in fact deep dynamical roots: it follows from the same basic dynamics as the “superconductivity” of the QCD vacuum, the chiral (χ -)symmetry breaking. These spin-isospin-zero diquarks are related to pions, as we will see below.

The most straightforward argument for deeply bound diquarks came from the bi-color ($N_c = 2$) theory: in it the scalar diquark is degenerate with pions. By continuity from $N_c = 2$ to 3, a trace of it should exist in real QCD¹².

Instantons create the following amusing *triality*: there are three attractive channels which compete: (i) the instanton-induced attraction in $\bar{q}q$ channel leading to χ -symmetry breaking. (ii) The instanton-induced attraction in qq which leads to color superconductivity. (iii) The *light-quark-induced* attraction of $\bar{I}I$, which leads to pairing of instantons into “molecules” and a Quark-Gluon Plasma (QGP) phase without *any* condensates.

At very high density we also can find *arbitrarily dilute instanton liquid*, as shown recently in [37]. The reason it cannot exist in vacuum or high T is that if instanton density goes below some critical value, the cannot be any condensate. (The system then breaks into instanton molecules or other clusters and chiral symmetry is restored.) However at high density the superconducting condensate can be created perturbatively as well (we mentioned it above) and there is no problem. The dilute instantons interact by exchanging very light η' (which would be massless without instantons): one can calculate effective Lagrangian, theta angle dependence etc.

Bi-color QCD: a very special theory One reason it is special (well known to to the lattice community): its fermionic determinant is *real* even for non-zero μ , which makes simulations possible. However the major interest in this theory is related the so called *Pauli-Gursey symmetry*. We have argued above that pions and diquarks appear at the same one-instanton level, and are so to say brothers. In bi-color QCD they becomes identical twins: due to the additional symmetry mentioned the diquarks are *degenerate* with mesons.

In particular, chiral symmetry breaking is done like this $SU(2N_f) \rightarrow Sp(2N_f)$, and for $N_f = 2$ the coset $K = SU(4)/Sp(4) = SO(6)/SO(5) = S^5$. Those 5 massless modes are pions plus the scalar diquark S and its anti-particle \bar{S} .

¹¹ for the same reason as lattice people cannot do it: the fermionic determinant is not real.

¹² Instanton-induced interaction strength in diquark channel is $1/(N_c - 1)$ of that for $\bar{q}\gamma_5 q$ one. It is the same at $N_c = 2$, zero for large N_c , and is exactly in between for $N_c = 3$.

Vector diquarks are degenerate with vector mesons, etc. Therefore, the scalar-vector splitting is in this case about twice the constituent quark mass, or about 800 MeV. It should be compared to binding in the “real” $N_c = 3$ QCD of only 200-300 MeV, and to zero binding in the large- N_c limit.

The corresponding sigma model describing this χ -symmetry breaking was worked out in[21]: for further development see[25]. As argued in [21], in this theory the critical value of the transition to Color Superconductivity is simply $\mu = m_\pi/2$, or zero in the chiral limit. The diquark condensate is just a rotated $\langle \bar{q}q \rangle$ one, and the gap is the constituent quark mass. Recent lattice works [28] display it in great detail, building confidence for other cases.

New studies reveal possible new crystalline phases. These phases still have somewhat debatable status, so I have not indicated them on the phase diagram.

Once again, there were two papers submitted by chance on the same day. The “Stony Brook” team[26] have found that a “chiral crystal” with oscillating $\langle \bar{q}q(x) \rangle$ (similar to Overhauser spin waves in solid state) can compete with the BCS 2-flavor superconductor at its onset, or $\mu \approx 400$ MeV. The proper position of this phase is somewhere in between the hadronic phase (with constant $\langle \bar{q}q \rangle$) and color superconductor.

The “MIT group”[27] have looked at the oscillating superconducting condensate $\langle qq(x) \rangle$, following earlier works on the so called LOFF phase in usual superconductors. They have found that it is appearing when the difference between Fermi momenta of different quark flavors become comparable to the gap. The natural place for it on the phase diagram is close to the line at which color superconductivity disappears because the gap goes to zero.

5 High Energy Collisions of hadrons and Heavy Ions

5.1 The Little Bang: AGS, SPS and now the RHIC era

Let me start with brief comparison of these two magnificent explosions: the Big Bang versus the Little Bang, as we call heavy ion collisions.

The expansion law is roughly the Hubble law in both, $v(r) \sim r$ although strongly anisotropic in the Little Bang. The Hubble constant tells us the expansion rate today: similarly radial flow tells us the final magnitude of the transverse velocity. The acceleration history is not really well measured. For Big Bang people use distance supernovae, we use Ω^- which does not participate at the late stages to learn *what was the velocity earlier*. Both show small dipole (quadrupole or elliptic for Little Bang) components which has some physics, and who knows maybe we will see higher harmonics fluctuations later on, like in Universe. As we will discuss below, in both cases the major puzzle is how this large entropy has been actually produced, and why it happened so early.

The major lessons we learned from AGS experiments ($E_{LAB} = 2 - 12AGeV$) are:

- (i) Strangeness enhancement over simple multiple NN collisions appear from very low energies, and heavy ion collisions quickly approach nearly ideal chemical equilibrium of strangeness.
- (ii) “Flows” of different species, in their radial,directed and elliptical form, are in this energy domain driven by collective potentials and absorptions: they are not really flows in hydro sense. All of them strongly diminish by the high end of the AGS region, demonstrating the onset of “softness” of the EoS. Probably it is some precursor of the QCD phase transition.

Several important lessons came so far from CERN SPS data:

- (i) Much more particle ratios have been measured there: overall those show surprisingly good degree of chemical equilibration: the chemical freeze-out parameters are tantalizingly close to the QGP phase boundary.
- (ii) Dileptons show that radiation spectral density is very different in dense matter compared to ideal hadronic gas. The most intriguing data are CERES finding of “melting of the ρ ”, which seem to be transformed into a wide continuum reaching down to invariant masses as low as 400 MeV. It puts in doubt “resonance gas” view of hadronic matter at these conditions. Intermediate mass dileptons studied by NA50 can be well described by thermal radiation with QGP rates.
- (iii)The impact parameter of J/ψ and ψ' suppression in PbPb collisions studied by NA50 collaboration shows rather non-trivial behavior. More studies are needed, including especially measurements of the open charm yields, to understand the origin and magnitude of the suppression.

However, during last several months those discussions have been overshadowed by a list of news from RHIC, Relativistic Heavy Ion Collider at Brookhaven National Laboratory. It had its first run in summer 2000 and reported recently at Quark Matter 2001 conference [29]: many details are discussed in Prof.M.Gylassy’s lectures.

A brief summary is as follows. These results have shown that heavy ions collisions (AA) at these energies significantly differ *both* from the pp collisions at high energies and the AA collisions at lower (SPS/AGS) energies. The main features of these data are quite consistent with the Quark-Gluon Plasma (QGP) (or Little Bang) scenario, in which entropy is produced promptly and subsequent expansion is close to adiabatic expansion of equilibrated hot medium.

(Let me mention here two other pictures of the heavy ion production, discuss prior to appearance of these data. One is the *string picture*, used in event generators like RQMD and UrQMD: they predicted effectively very soft EoS and elliptic flow decreasing with energy. The other one is *pure minijet scenario*, in which most secondaries would come from independently fragmenting minijets. If so, there are basically no collective phenomena whatsoever.)

Already the very first multiplicity measurements reported by PHOBOS collaboration [54] have shown that particle production per participant nucleon is no longer constant, as was the case at lower (SPS/AGS) energies.

This new component may be due to long-anticipated $pQCD$ processes, leading to perturbative production of new partons. Unlike high p_t processes resulting in visible jets, those must be undetectable “*mini-jets*” with momenta $\sim 1 - 2 \text{ GeV}$. Production and decay of such *mini-jets* was discussed in Refs [55], also this scenario is the basis of widely used event generator HIJING [51]. Its crucial parameter is the *cutoff scale* p_{min} : if fitted from pp data to be $1.5\text{-}2 \text{ GeV}$, it leads to predicted mini-jet multiplicity $dN_g/dy \sim 200$ for central AuAu collisions at $\sqrt{(s)} = 130 \text{ AGeV}$. If those fragment independently into hadrons, and are supplemented by “soft” string-decay component, the predicted total multiplicity was found to be in good agreement with the first RHIC multiplicity data. Because partons interact perturbatively, with their scattering and radiation being strongly peaked at small angles, their equilibration is expected to be relatively long [56]. However, new set of RHIC data reported in [29] have provided serious arguments *against* the mini-jet scenario, and point toward quite rapid entropy production rate and early QGP formation.

(i) If most of mini-jets fragment independently, there is no *collective phenomena* such as transverse flow related with the QGP pressure. However, it was found that those effects are very strong at RHIC. Furthermore, STAR collaboration have observed very robust *elliptic flow* [39], which is in perfect agreement with predictions of hydrodynamical model [46,45] assuming equilibrated QGP with its full pressure $p \approx \epsilon/3$ above the QCD phase transition. This agreement persists to rather peripheral collisions, in which the overlap almond-shaped region of two nuclei is only a couple fm thick. STAR and PHENIX data on spectra of identified particles, especially p, \bar{p} , indicate spectacular radial expansion, also in agreement with hydro calculations [46,45]. (ii) Spectra of hadrons at large p_t , especially the π^0 spectra agree well with HIJING for peripheral collisions, but show much smaller yields for central ones, with rather different, (exponential-shaped) spectra. It means long-anticipated “*jet quenching*” at large p_t is seen for the first time, with a surprisingly large suppression factor $\sim 1/5$. Keeping in mind that jets originating from the surface outward cannot be quenched, the effect seem to be as large as it can possibly be. For that to happen, the outgoing high- p_t jets should propagate through matter with parton population larger than the abovementioned minijet density predicted by HIJING.

(iii) Curious interplay between collective and jet effects have also been studied by STAR collaboration, in form of elliptic asymmetry parameter $v_2(p_t)$. At large transverse momenta $p_t > 2 \text{ GeV}$ the data depart from hydro predictions and levels off. When compared to predictions of jet quenching models worked out in [57], they also indicate gluon multiplicity several times larger than HIJING prediction, and are even consistent with its maximal possible value evaluated from the final entropy at freeze-out, $(dN/dy)_\pi \sim 1000$.

5.2 Collective flows and EoS

If we indeed have produced excited matter (rather than just a bunch of partons which fly away and fragment independently), we expect to see certain collective phenomena. Ideally, those should be quantitatively reproduced by relativistic hydrodynamics which is basically just local energy-momentum conservation plus the EoS we know from the lattice and models.

The role of the QCD phase transition in matter expansion is significant. QCD lattice simulations [43] show approximately 1st order transition. Over a wide range of energy densities $e = .5 - 1.4 \text{ GeV}/\text{fm}^3$ the temperature T and pressure p are nearly constant. So the ratio of pressure to energy density, p/e , decreases till a minimum at particular energy density $e_{sp} \approx 1.4 \text{ GeV}/\text{fm}^3$, known as the *softest point* [44]. Near e_{sp} small pressure gradient can not effectively accelerate the matter and the evolution stagnates. However when the initial energy density is well above the QCD phase transition region, $p/e \approx 1/3$, and this pressure drives the collective motion. The energy densities reached at time $\sim 1 \text{ fm}/c$ at SPS ($\sqrt{s}_{NN} = 17 \text{ GeV}$) and RHIC ($\sqrt{s}_{NN} = 130 \text{ GeV}$) are about 4 and $8 \text{ GeV}/\text{fm}^3$, respectively. We found that at RHIC conditions we are in the latter regime, and matter accelerates to $v \sim .2c$ before entering the soft domain. Therefore by freeze-out this motion changes the spatial distribution of matter dramatically: e.g. as shown in [38] the initial almond-shape distribution 10 fm/c later looks like two separated shells, with a little “nut” in between.

The simplest way to see hydro expansion is in spectra of particles: on top of chaotic thermal distributions $\sim \exp(-m_t/T)$, $m_t^2 = p_t^2 + m^2$ one expect to see additional broadening due to hydro outward motion. This effect is especially large if particles are heavy, since flow with velocity v add momentum mv .

Derek Teaney [46] have developed a comprehensive Hydro-to-Hadrons (H2H) model combines the hydrodynamical description of the initial QGP/mixed phase ($e > .5 \text{ GeV}/\text{fm}^3$) stages, where hadrons are not appropriate degrees of freedom, with a hadronic cascade RQMD for the hadronic stage. In this way, we can include different EoS displaying properties of the phase transition, and also incorporate complicated final state interaction at freeze-out. The set of EoS used is shown in Fig.10.

Radial flow is usually characterized by the slope parameter T: each particle spectra are fitted to the form $dN/dp_t^2 dy \sim \exp(-m_t/T)$, $m_t^2 = p_t^2 + m^2$. Although we denoted the slope by T, it is *not* the temperature: it incorporates random thermal motion and collective transverse velocity. The SPS NA49 slope parameters for pion and protons are shown in Fig. 11(a). Parameter T grows with particle multiplicity due to increased velocity of the radial flow. Furthermore, the rate of growth depends on the EoS: the softer it is, the less growth. The SPS NA49 data correspond to two data points (our fits to spectra) favor the (relatively stiff) LH8 EoS. (Details of the fit, discussion of

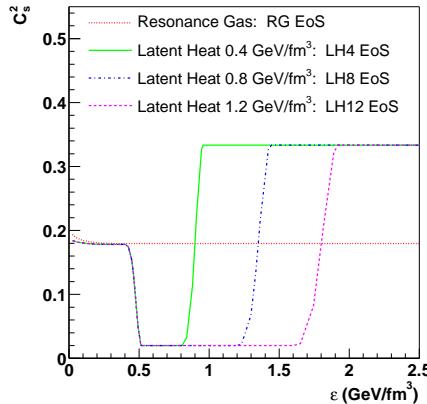


Fig. 10. The EoSs in form squared speed of sound $c^2 = dp/d\epsilon$ with variable Latent Heats $.4 GeV/fm^3$, $.8 GeV/fm^3$,.. labeled as LH4, LH8,..versus the energy density.

the b-dependence etc see in [46].) It is very important to get these parameters for RHIC, especially for heavy secondaries like nucleons and hyperons.

For *non-central* collisions the overlap region in the transverse plane has an elliptic, “almond”, shape, and larger pressure gradient force matter to expands preferentially in the direction of the impact parameter [41]. Compared to radial flow, the elliptic flow is formed earlier, and therefore it measures the early pressure. The *elliptic flow* is quantified experimentally by measuring the azimuthal distributions of the produced particles and calculating the elliptic flow parameter $V_2 = \langle \cos(2\phi) \rangle$ where ϕ angle is measured with respect to the impact parameter direction, around the beam axis. It appears due to the elliptic *spatial* deformation of the overlap region in the nucleus-nucleus collision, quantified by its eccentricity $\epsilon_2 = \langle y^2 - x^2 \rangle / \langle x^2 + y^2 \rangle$, usually calculated in Glauber model. Since the effect (v_2) is proportional to the cause (ϵ_2), the ratio v_2/ϵ_2 does not have strong dependence on the impact parameters b , and this ratio is often used for comparison. (We would not do that below, in the detailed comparison to data, because $\epsilon_2(b)$ is not directly measured.

In figure 11(b) the elliptic flow of the system is plotted as a function of charged particle multiplicity at an impact parameter of 6 fm. Before discussing the energy dependence, let us quantify the magnitude of elliptic flow at the SPS. Ideal relativistic hydrodynamics used in earlier works [41,45] generally over-predicts elliptic flow by about factor 2. Example of such kind is indicated by a star in figure 11(b): it is our hydro result (with LH8 EoS) which has been followed hydrodynamically till very late stages, the freeze-out temperature $T_f = 120 MeV$. By switching to hadronic cascade at late stages, we have more appropriate treatment of resonance decays and re-scattering

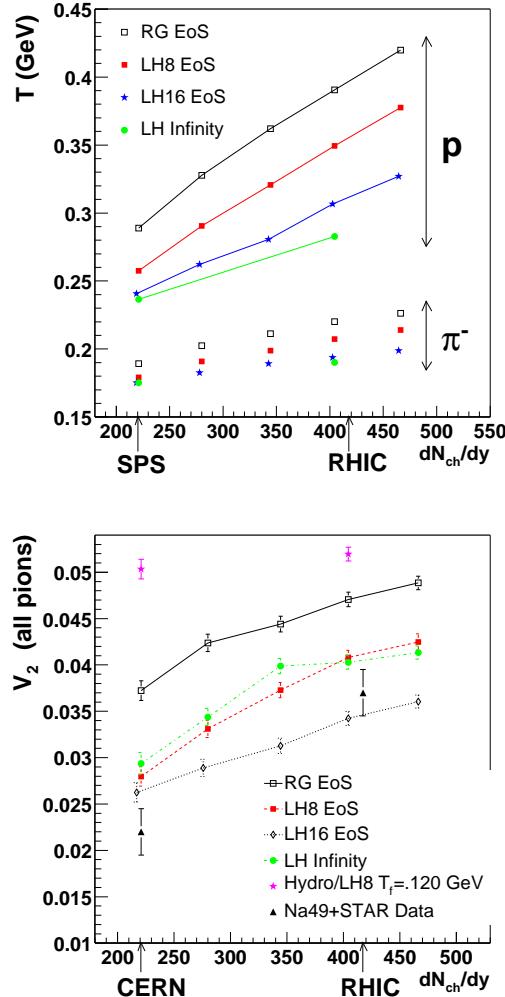


Fig. 11. The transverse mass slope T (a) and elliptic flow parameter V_2 (b) versus midrapidity ($y=0$) charged particle multiplicity, for AuAu collisions with $b=6$.

rate, and so one can see that it significantly reduces V_2 , to the range much closer to the data points.

One might think that one can also do that by simply taking *softer* EoS, e.g. increasing the latent heat. However, it only happens till LH16 and then v_2 start even slightly increase again. The explanation of this non-monotonous behavior is the interplay of the initial “QGP push” for stiffer EoS, with longer time for hadronic stage available for softer EoS. We cannot show here

details, but it turns out that a given (experimental) V_2 value can correspond to *two different solutions*, one with earlier push and another with the later expansion dominating. Coincidentally, STAR data point happen to be right at the onset of such a bifurcation, close to LH16. The *multiplicity dependence* of V_2 appears simple from figure ??(b): all curve show growth with about the same rate. Note however that such growth of V_2 from SPS to RHIC (first predicted in [49] where our first preliminary results has been shown) Is not shared by most other models. In particular, *string-based* models like UrQMD predicts a decrease by a factor of ≈ 2 [50]. It happens because strings produce no transverse pressure and so the effective EoS is super-soft at high energies. Models based on *independent parton scattering and decay* (such as HIJING) also predict basically vanishing (or slightly negative)[51] V_2 .

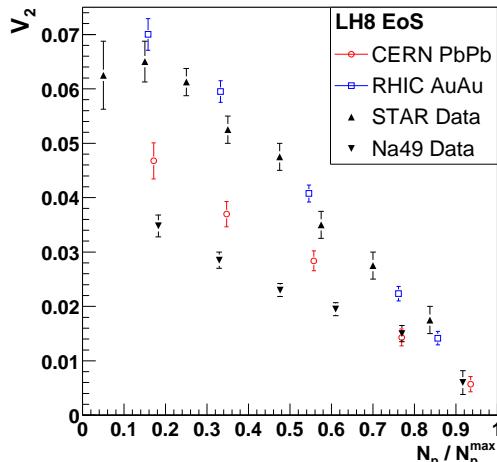


Fig. 12. v_2 versus impact parameter b , described experimentally by the number of participant nucleons, for RHIC STAR and SPS NA49 experiments. Both are compared to our results, for EoS LH8.

In Fig.12 we show how our results compare with data as a function of impact parameter. One can see that the agreement becomes much better at RHIC. Furthermore, one may notice that deviation from linear dependence we predict becomes visible at SPS for more peripheral collisions with $N_p/N_p^{max} < 0.6$ or so, while at RHIC only the most peripheral point, with $N_p/N_p^{max} = 0.05$ show such deviation. This clearly shows that hydrodynamical regime in general works much better at RHIC.

In summary, the flow phenomena observed at RHIC are stronger than at SPS. It is in complete agreement with the QGP scenario. All data on elliptic and radial flow can be nicely reproduced by the H2H model. Furthermore, we are able to restrict the EoS, to those with the latent heat about $.8 \text{ GeV}/\text{fm}^3$.

5.3 How QGP happened to be produced/equilibrated so early?

One possible solution to the puzzle outline above can be a *significantly lower cutoff scale* in AA collisions, as compared to $p_{min} = 1.5 - 2 \text{ GeV}$ fitted from the pp data. That increases perturbative cross sections, both due to smaller momenta transfer and larger coupling constant. As I argued over the years, the QGP is a new phase of QCD which is *qualitatively different* from the QCD vacuum: therefore the cut-offs of pQCD may have entirely different values and be determined by different phenomena. Furthermore, since QGP is a plasma-like phase which screens itself perturbatively [40], one may think of a cut-offs to be determined *self-consistently* from resummation of perturbative effects. These ideas known as *self-screening* or *initial state saturation* were discussed in Refs. [56]. Although the scale in question grows with temperature or density, *just above T_c* it may actually be *smaller* than the value 1.5-2 GeV we observe in the vacuum. Its first experimental manifestation may be dropping of the so called “duality scale” in the observed dilepton spectrum, see discussion in [60].

Another alternative to explain large gluon population at RHIC would be an existence of more rapid multi-gluon production processes. Let us consider an alternative *non-perturbative* scenario based entirely on non-perturbative processes involving *instantons* and *sphalerons* [59]. But before we do that, we have to take a look at hadronic collisions and briefly review few recent papers on the subject.

At $s > 10^3 \text{ GeV}^2$ hadronic cross sections as $\bar{p}p, pp, \pi p, Kp, \gamma N$ and even $\gamma\gamma$ slowly grow with the collision energy s. This behavior can be well parameterized by *soft Pomeron* phenomenology, but we will only use its logarithmically growing part

$$\sigma_{hh'}(s) = \sigma_{hh'}(s_0) + \log(s/s_0) X_{hh'} \Delta + \dots \quad (38)$$

ignoring both the higher powers of $\log(s)$ and decreasing Regge terms. We will use those two parameters from PDG-2000 recent fits, the intercept and its coefficient in $pp, \bar{p}p$ collisions, $\Delta = \alpha(0) - 1 = 0.093(2)$, $X_{NN} = 18.951(27) \text{ mb}$. Note a qualitative difference between constant and logarithmically growing parts of the cross section. The former can be explained by prompt color exchanges, as suggested by Low and Nussinov long ago. It nicely correlates with flux tube picture of the final state. section.) The *growing* part of the cross section cannot be generated by t-channel color exchanges and is associated with processes promptly producing some objects, with $\log(s)$, coming from the longitudinal phase space. In pQCD it is *gluon* production,

by processes like the one shown in Fig.13(a). If iterated in the t-channel in ladder-type fashion, the result is approximately a BFKL pole [61]. Although the power predicted is much larger than Δ mentioned, it seems to be consistent with much stronger growth seen in hard processes at HERA: thus it is therefore sometimes called the “hard pomeron”. The physical origin of cross section growth remains an outstanding open problem: neither the perturbative resummations nor many non-perturbative models are really quantitative. It is hardly surprising, since scale at which soft Pomeron operates (as seen e.g. from the Pomeron slope $\alpha'(0) \approx 1/(2\text{ GeV})^2$) is also the “substructure scale” mentioned above.

Recent application of the instanton-induced dynamics to this problem have been discussed in several papers [62]. Especially relevant for this Letter are two last works which use insights obtained a decade ago in discussion of instanton-induced processes in electroweak theory [35], and the growing part of the hh cross sections were ascribed to multi-gluon production via instantons, see Fig.13(b). Among qualitative features of this theory is the explanation of why no odderon appears (instantons are SU(2) objects, in which quarks and antiquarks are not really distinct), an explanation of the small power Δ (it is proportional to “instanton diluteness parameter” $n\rho^4$ mentioned above), the small size of the soft Pomeron (governed simply by small size of instantons $\rho \sim 1/3\text{ fm}$). Although instanton-induced amplitudes contain small “diluteness” factor, there is no extra penalty for production of new gluons: thus one should expect instanton effects to beat perturbative

amplitudes of sufficiently high order. This generic idea is also behind the present work, dealing with prompt multi-gluon production.

Technical description of the process can be split into two stages. The first (at which one evaluates the probability) is the motion *under the barrier*, and it is described by Euclidean paths approximated by instantons. Their interaction with the high energy colliding partons results in some energy deposition and subsequent motion *over the barrier*. At this second stage the action is real, and the factor $\exp(iS)$ does not affect the probability, and we only need to consider it for final state distributions. The relevant Minkowski paths start with configurations close to the QCD analogs of electroweak *sphalerons* [63], static spherically symmetric clusters of gluomagnetic field which satisfy the Yang-Mills equations. (Those can be obtained from known electroweak solutions in the limit of infinitely large Higgs self-coupling.) Their mass in QCD is

$$M_{sph} \approx \frac{30}{g^2(\rho)\rho} \sim 2.5\text{ GeV} \quad (39)$$

Since those field configurations are close to classically unstable saddle point at the top of the barrier, they roll downhill and develop gluoelectric fields. When both become weak enough, solution can be decomposed into perturbative gluons. This part of the process can also be studied directly from classical Yang-Mills equation: for electroweak sphalerons it has been done in Refs[64],

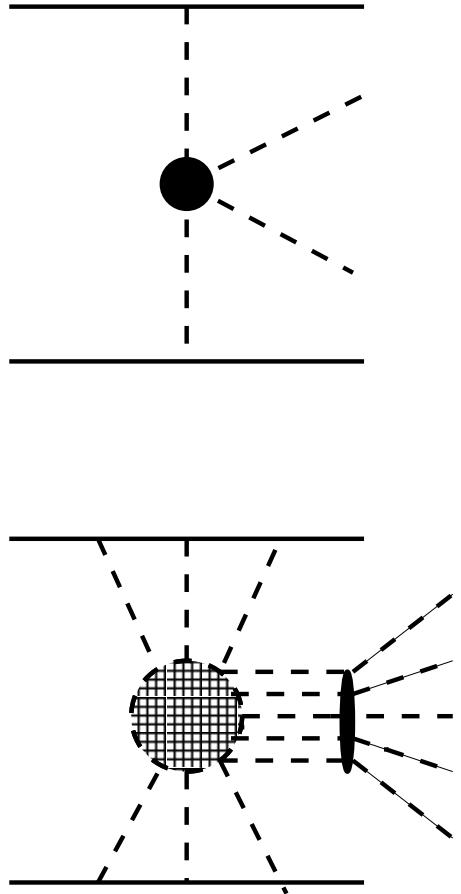


Fig. 13. (a) A typical inelastic perturbative process: two t-channel gluons collide, producing a pair of gluons; (b) Instanton-induced inelastic process incorporate collisions of multiple t-channel gluons with the instanton (the shaded circle), resulting in multi-gluon production. The intermediate stage of the process, indicated by the horizontal dashed lines, corresponds to a time when outgoing glue is in the form of coherent field configuration - the *sphaleron*. Since this part of the process corresponds to motion above the barrier, it does not enter the calculation of the cross section, but is only needed for prediction of the inclusive spectra, multiplicities etc.

calculation for its QCD version is in progress [65]. While rolling, the configurations tend to forget the initial imperfections (such as a non-spherical shapes) since there is only one basic instability path downward: so the resulting fields should be nearly perfect spherical expanding shells. Electroweak sphalerons decay into approximately 51 W,Z,H quanta, of which only about 10% are Higgses, which carry only 4% of energy. Ignoring those, one can estimate mean gluon multiplicity per sphaleron decay, by simple re-scaling

of the coupling constants: the result gives 3-4 gluons. Although this number is not large, it is important to keep in mind that they appear as a coherent expanding shell of strong gluonic field.

It has been suggested in [59] that if sphaleron-type object are copiously produced, with or instead of $p \sim 1\text{GeV}$ minijets, they may significantly increase the entropy produced and speed up the equilibration process.

Acknowledgments. It is a pleasure to thank the organizers of the school for their invitation: it was indeed a very interesting and successful one. This work was supported in parts by the US-DOE grant DE-FG-88ER40388.

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